TUTORIAL SETS 37, 38, 39, 40 AND 41 COMBINED

The basic information for these tutorial sets is part of a booklet produced by James E. Cross for the Mount Pilgrim Baptist Church Tutorial Program in May 2002.

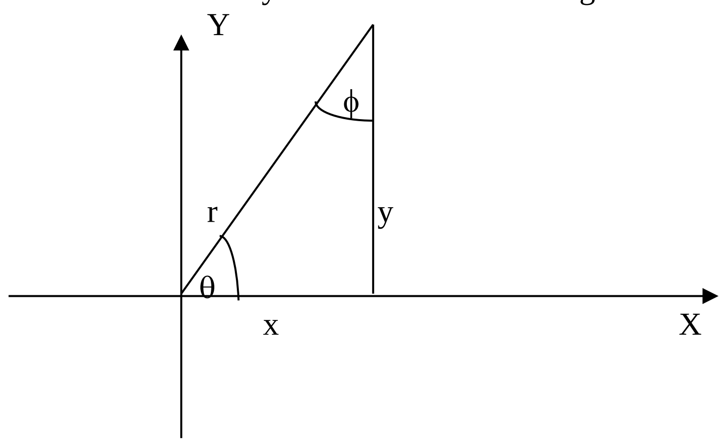
Tutorial Set 37- An Introduction To The Six Trigonometric Functions: Part 1

I. Introduction

The word trigonometry means triangle measurement. It is a branch of mathematics that relates angles and distances. Most of the trigonometric functions involve right triangles. This branch of mathematics is widely used in science and engineering. In building a house, the walls should be upright, that is, set at 90 degrees. Plumbers are required to have the sewerage lines sloping at some minimum angle such that the sewerage will drain down into the sewerage system. Other examples will be given.

II. The Right Triangle and the Six Trigonometric Functions

Consider a right triangle placed in the coordinate system as shown in this figure.



This is a right triangle. The side r is the hypotenuse since it is opposite the right angle (90 degrees). The sides x and y are the legs. We have from Pythagorean theorem:

$$r^2 = x^2 + y^2 \text{ or } r = \sqrt{x^2 + y^2}$$

Using the angle θ (theta) as shown, we define six trigonometric functions as follows in terms of the ratios of two of the sides:

$\sin(\theta) = y/r$ (the sine) or the ratio of the opposite side to the hypotenuse.
$\cos(\theta) = x/r$ (the cosine) or the ratio of the adjacent side to the hypotenuse.
$\tan(\theta) = y/x$ (the tangent) or the ratio of the opposite to the adjacent side.
$\sec(\theta) = r/x$ (secant) or the ratio of the hypotenuse to the adjacent side.
$\csc(\theta) = r/y$ (cosecant) or the ratio of the hypotenuse to the opposite side.
$\cot(\theta) = x/y$ (cotangent) or the ratio of the adjacent to the opposite side.

Notice that cosecant is the reciprocal of sine, while from the name you might expect it to be the reciprocal of cosine!

We will give some examples using Pythagorean theorem and the six trigonometric functions and then give you an opportunity to solve some problems. You can check your answers using a scientific calculator or by using the Excel software on your computer.

Example 1. Assume that x on the above diagram is 6 ft as a distance being required between two persons because of the Coronavirus. Also, assume that the distance y is 8 feet. What is the distance r?
Solution: r = square root of $(x^2 + y^2)$ = square root of $(6^2 + 8^2)$ = square root of (36 + 64) = square root of (100) = 10. So, the three sides are
x = 6 ft
y = 8 ft
r = 10 ft.

Example 2. What is the value for $\sin(\theta)$ for the values given in Example 1?
Solution: $\sin(\theta) = y/r = 8/10 = .8$
Note: We will later show that the value of the angle can be found using a calculator or Excel software.

Work the following problems:

1. Given the values of x = 6 ft and y = 8 ft, find the value of $\cos(\theta)$.
2. Given the values of x = 6 ft and y = 8 ft, find the value of $\tan(\theta)$.
3. Given the values of x = 6 ft and y = 8 ft, find the value of $\csc(\theta)$.
4. Given the values of x = 6 ft and y = 8 ft, find the value of $\sec(\theta)$.
5. Given the values of x = 6 ft and y = 8 ft, find the value of $\cot(\theta)$.

Tutorial Set 38- Angles In Degrees And Radians: Part 1I

It is important to understand that the value of angles can be expressed in degrees or radians. A calculator or Excel software will want to know if the angle is in degrees or radians. The relationship between angles and degrees can be shown by considering that there are 360 degrees in a circle. Concerning radians, the circumference of a circle when divided by the diameters has to be shown to be C/D = 3.1416 radians where C is the circumference and D is the diameter. The value 3.1416 is noted by the Greek letter pi, π. That is $\pi = 3.1416$. (When given as a fraction, 22/7 is usually used.) In that the diameter is twice the radius R, we also have $C/R = 2\pi = 6.2832$ radians.

A circle contains 360 degrees or 2π radians. Thus,
2π radians = 360 degrees. Then 1 radian = 360/6.2832 = 57.3 degrees.

To change degrees to radians, divide the number of degrees by 57.3.
To change radians to degrees, multiply the number of radians by 57.3.
The relationship is often expressed in terns of π. As example, 180 degrees = π radians.
To show this, we can say that if 360 degree = 2π radians, then for y degrees, we use the ratio
$360/y = 2\pi/x$. Given y in degrees, we have
$x = (2\pi)y/360$ radians.

As an example, to express 180 degrees in terms of radians, let y = 180 to get
$x = (2\pi)180/360 = \pi$ radians.
Or, 180 degrees = π.

Some of the angles used very often in examples and in practice are 30 degree, 45 degree, 90 degrees, 135 degrees, 180 degrees, 270 degrees, 270 degree and 360 degrees (each being some multiple of 45 degrees). For the above example, 180 degrees was expressed in terms of π.

Perform the following:

1. Express 45 degrees in terms of π.
2. Express 90 degrees in terms of π.
3. Express 135 degrees in terms of π.
4. Express 270 degrees in terms of π.
5. Express 30 degrees in terms of π.

Tutorial Set 39 – The Inverse Of Trigonometric Functions: Part 1II

The inverse of a trigonometric function such as sin(θ) is denoted by $\sin^{-1}(\theta)$. In particular, if sin(θ) = k, then $\theta = \sin^{-1}(k)$. This is the same for the other trigonometric functions. For a carpenter cutting a piece of board and it is to be a given angle, if he has determined one side, he might want to calculate the length of the other side to give a certain angle. On another occasion, he might be given the ratio of the sides and might need to determine the angle. So for trigonometric problems, in some cases, the angle and one side might be given and the other sides are to be found. Or the sides might be given and the angle is to be found. For the previous section, it was assume that the angle was given. Now, consider that the ratio of certain sides is given and the angle is to be found.

Example: Find the angle θ if for the given triangle the distance r (the hypotenuse) is 10 feet and the side y, opposite the angle Θ, is 6 feet long.
Solution: We can use the sine of the angle as sin(θ) = y/r = 6/10 or sin(θ) = .6.
Then $\theta = \sin^{-1}(.6)$. Using a scientific calculator or Excel, we find that θ = 38.87 degrees.

Note: In addition to Excel, scientific languages for computers such as C and MATLAB also provide the trigonometric functions. Computers assume that the angle is given in radians. If we wanted to find the sine of 45 degrees we would express this as 3.1416/4 or .785 radians. The inverse sine is typically called asin (the term arc being prefixed). The other trigonometric functions have similar expressions. Thus, for the above problem, we would ask the computer to find asin(.6). C and MATLAB software programs must be purchase if you wish to use it whereas Xcel is part of the Microsoft Office package that is more commonly found on computers.

To solve the above example using Excel, from Home on the computer, click on Formulas and select math & trig. Type in .6, hit enter to get .6435. This is the angle in radians. Multiply by 57.3 to get the angle as 36.87 degrees.

Work the following problems using a scientific calculator or Exel using x, y and r for the given triangle:

1. Find the angle θ if for the given triangle the distance r (the hypotenuse) is 10 feet and y, the side opposite the angle Θ, is 8 feet long. (The angle is to be in degrees for all problems).
2. Find the angle θ if for the given triangle the distance r (the hypotenuse) is 10 feet and x, the side adjacent the angle Θ, is 6 feet long.
3. Find the angle θ if for the given triangle the distance r (the hypotenuse) is 10 feet and x, the side adjacent the angle Θ, is 8 feet long.
4. Find the angle θ if for the given triangle the distance y is 8 feet and the distance x is 6 feet long.
5. Find the angle θ if for the given triangle the distance y is 6 feet and the distance x is 8 feet long.

Tutorial Set 40 – Problems With Angles And Distance Using Trigonometric Functions: Part IV

Example. We wish to find the height of a tree. We come back 80 feet from the tree and find that an angle between the ground and the top of the tree is 60 degrees. Since the tangent is equal to the opposite side, y, over the adjacent side, x, and the adjacent side, x, is given as 80, use
$\tan(\Theta) = y/x$ where $\Theta = 60$ degrees. From a scientific calculator find
$\tan(60) = 1.732$. Then $1.732 = y/80$ and $y = 1.732(80) = 138.56$ feet.

To use Xcel, change degrees to radians and after selecting tan from the math & trig functions, type in 60/57.3 (to change to radians) and get 1.7317. Multiply by 80 to get y = 138.539.

Work the following:

1. You are to find the height of a tree where you come back 90 feet from the tree and found that an angle between the ground and the top of the tree is 30 degrees.

2. You are to find the height of a tree where you come back 100 feet from the tree and found that an angle between the ground and the top of the tree is 45 degrees.

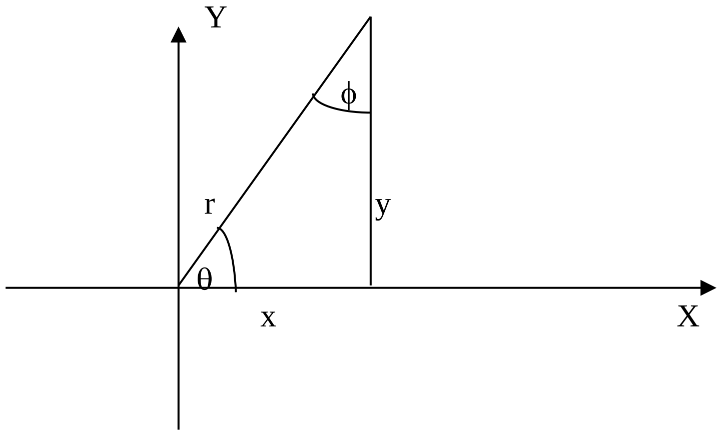
3. In Electrical Engineering, it is often required to find the electrical current flowing in a wire. Circuits that have coils of wire will have inductance reactance designated as Xl and resistance designated as R. The combination of these two is called impedance designated as Z. The relationship is $R = Z\cos(\Theta)$ and $Xl = Z\sin(\Theta)$. Find R and Xl given that $Z = 100$ and $\Theta = 60$ degrees.

4. An airplane is approaching the airport. It is 3 miles high, y. A radar beam shows a distance to the plane, r , of 5 miles. How far is the airplane (x) from the airport? What was the angle from the ground to the airplane formed by the radar beam?

Tutorial Set 41 – Additional Relationships With Angles Considering Trigonometric Functions: Part V

I. Some Trigonometric Identities

Consider a right triangle placed in the coordinate system as shown in this figure.



A. $\sin^2(\theta) + \cos^2(\theta) = 1$

Proof: Since $\sin(\theta) = y/r$ and $\cos(\theta) = x/r$ for a right triangle. Then

$\sin^2(\theta) + \cos^2(\theta) = (y/r)^2 + (x/r)^2 = (y^2 + x^2)/r^2 = r^2/r^2 = 1$

(Note that we used Pythagorean theorem, $r^2 = x^2 + y^2$).

B. $\sin(-\theta) = -\sin(\theta)$

Proof: As seen by this figure, $\sin(\theta) = y/r$ and $\sin(-\theta) = -y/r$.

C. $\cos(\theta) = \cos(-\theta)$

Proof: As seen by this figure, $\cos(\theta) = x/r$. But since r is always positive,

and x is common to both θ and $-\theta$, $\cos(-\theta) = x/r$ also.

II. Summary of Widely Used Trigonometric Values

Angle θ, In Degrees	Function	Value
0	sine(θ)	0.0
	cos(θ)	1.0
	tan(θ)	0.0

30	sine(θ)	.50
	cos(θ)	.867
	tan(θ)	.577
36.9	sine(θ)	.60
	cos(θ)	.80
	tan(θ)	.75
45	sine(θ)	.707
	cos(θ)	.707
	tan(θ)	1.0
53.1	sine(θ)	.80
	cos(θ)	.60
	tan(θ)	1.33
60	sine(θ)	.867
	cos(θ)	.50
	tan(θ)	1.732
90	sine(θ)	1.0
	cos(θ)	0.0
	tan(θ)	∞

Work the following problems:

1. How many radians are there in 30 degrees?

2. Consider the identity sin(a+b) = sin(a)cos(b) + cos(a)sin(b)
   Let a = 45 degrees and b = 45 degrees and show that this identity is true.

3. cos(a + b) = cos(a)cos(b) - sin(a)sin(b)
   Let a = 30 degrees and b = 15 degrees and show that this identity is true.

4. Let a = 30 degrees and show that $\sin^2(\theta) + \cos^2(\theta) = 1$.