

Tutorial Sets 67, 68, 69, 70 and 71: Areas and Circumference of Figures

These exercises will show equations for calculating the circumference and area of triangles, squares, rectangles, trapezoids and circles.

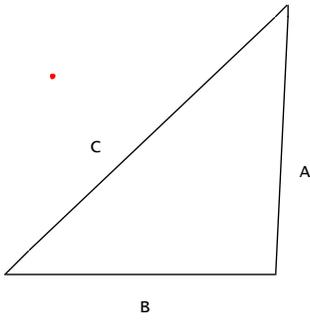


Figure 1

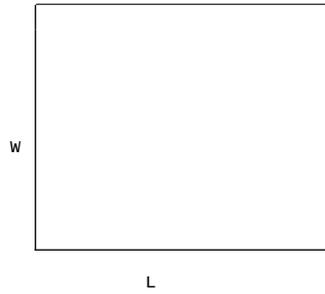


Figure 2

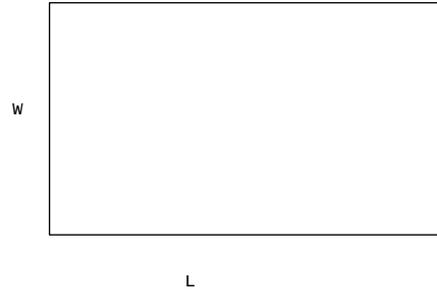


Figure 3

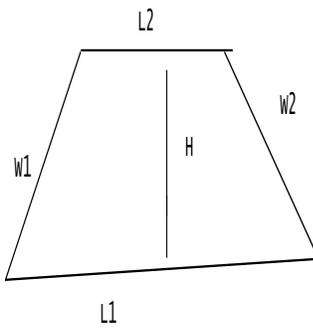


Figure 4

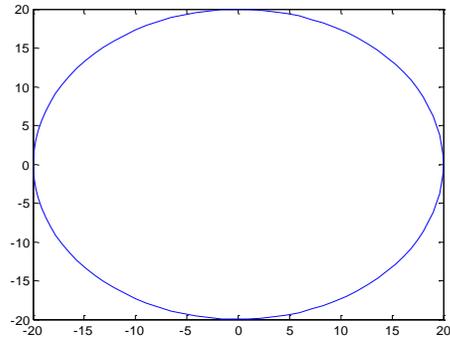


Figure 5

Tutorial Set 67- Triangles

Consider Figure 1. This is an example of a right triangle in that it has a 90 degree angle. The circumference of a triangle is equal to the sum of the length of its sides. We will call this F. Then for the figure shown, $F = A + B + C$.

The area for a triangle is equal to one half the base (b) times the height (h). That is
Area = (1/2)bh

For figure 1, the base is shown as B and the height is shown as A.

Example 1. Find the area of a right triangle that has a base of 20 inches and a height of 12 inches.

Solution: Area = $(1/2)(20)(12) = 120$ square inches.

You are encouraged to consult other source, especially Youtube, to see presentations for triangles that are not right triangles. In particular, you might view a presentation on Youtube that presents Horn's formula for any type of triangle. It is as follows: Given a triangle with the length of the sides being a, b, and c, let $S = (a + b + c)/2$. Then, the area is given as

Area = square route of [(S)(S-a)(S-b)S-c]

Example 2: Again, consider Figure 1. Let $B = 4$, and $A = 3$. Then Area = $(1/2)(4)(3) = 6$.

We were not given the side C but since this is a right triangle, from Pythagorean theorem, we can find $C = \text{square route of } (A^2 + B^2) = \text{square route of } (25) = 5$.

Then from Horn's formula, $S = (3 + 4 + 5)/2 = 6$.

$S(S-a)(S-b)(S-c) = (6)(6-3)(6-4)(6-5) = (6)(3)(2)(1) = 36$

Then, Area = square route of $36 = 6$, the same as we obtain above.

Note that given the length of the 3 sides, using Horn's formula, we can find the area of any triangle, no matter what the shape.

Your Exercise:

1. Find the area of the right triangle shown in Figure 1 if the base = 6 inches and height = 8 inches using the simple formula.
2. For problem 1, above, noting that B is the base and A is the height, find C and then use Horn's formula to find the area.
3. Use Horn's formula to find the area of a triangle with the length of the three sides given as 12, 16 and 25 inches.
4. For a right triangle, what is its height if the area = 800 square inches and the base is 20 inches?

Tutorial Set 68- Rectangles

We will first consider Figure 3, a rectangle, and come back to Figure 2, the square. A rectangle has four sides with the two opposite sides being parallel and of equal length. Also, all four of the angles are right angles (or 90 degree angles). The sides for Figure 3 are labeled l for length and w for width. Two sides will have the length l and two sides will have the width w . The circumference of a rectangle is equal to the sum of the length of its sides which is $2l$ plus $2w$. That is, the

$$\text{Circumference} = 2l + 2w = 2(l + w).$$

The area for a rectangle is equal to the product of the length and the width. That is

$$\text{Area} = (l)(w)$$

Example. Find the circumference and the area of a rectangle with length of 12 inches and width of 4 inches.

Solution: Circumference = $2(12 + 4) = 2(16) = 32$ inches.

The Area = $(12)(4) = 48$ square inches.

Your Exercise:

1. Find the circumference of a rectangle with length of 20 inches and width of 5 inches.
2. Find the area of a rectangle with length of 20 inches and width of 5 inches.
3. What is the length of the sides of a rectangle if the area = 400 square inches and the width is 10 inches?

Tutorial Set 69- Squares

Consider Figure 2. The square is a special rectangle in that all the sides of a square are equal. The circumference of a square is equal to the sum of the length of its sides which is 4 times the side. The area for a square is equal to the square of the length of the side (l). That is

$$\text{Area} = l^2$$

Example. Find the circumference and the area of a square with the length of 5 inches.

Solution: Circumference = $(4)(5) = 20$ inches. Area = $5^2 = 25$ inches.

Your Exercise:

1. Find the circumference of a square with sides of 8 inches.
2. For the area of a square with sides of 8 inches.
3. What is the length of the sides of a square if the area = 100 square inches?

Tutorial Set 70 – Trapezoids

A trapezoid is shown in figure 4. It differs from a rectangle in that whereas two of the sides are parallel, the other two sides are not parallel and are not necessarily equal. As a result, the two parallel sides are not of equal length. We will consider both of the parallel sides as bases and will call them base 1 (b1) and base 2 (b2). (They are labelled L1 and L2 but change it to b1 and b2.)

The circumference of a trapezoid is equal to the sum of the length of its sides. If the sides are labeled b1, b2, w1 and w2, then

$$\text{Circumference} = b1 + b2 + w1 + w2$$

We could divide the trapezoid into a rectangle and two triangles with height of h (as shown). The area could then be shown to be one half the sum of the two bases times the height. That is

$$\text{Area} = (1/2)[b1 + b2]h$$

Example. Find the area of a trapezoid with base 1 being 20 inches, base 2 being 14 inches and the height being 6 inches.

Solution: The Area = $(1/2)[20 + 14]6 = (3)(34) = 102$ square inches.

Your Exercise:

1. Find the area of a trapezoid with base 1 being 10 inches, base 2 being 7 inches and the height being 12 inches.
2. Find the area of a trapezoid with base 1 being 40 inches, base 2 being 20 inches and the height being 8 inches.
3. What is the height of a trapezoid if the area = 200 square inches with base 1 being 30 inches and base 2 being 10 inches?

Tutorial Set 71- Circles

A circle as shown in Figure 5 will have all points on the circle the same distance from the center. That distance is called the radius, R . The distance from one side of a circle to the other side going through the center is twice the radius and is called the diameter, D . The circumference or distance around the circle is given as π times the diameter. The value of π is 3.14 or approximately $22/7$. Since the diameter is 2 times the radius, the circumference is also given as 2π times the radius. That is

$$\text{Circumference} = \pi D = 2\pi R.$$

The area is given as π times the radius square. That is

$$\text{Area} = \pi R^2$$

Example. Find the circumference of a circle and the area of a circle with radius of 10 inches.

Solution: Circumference = $(2)(3.14)(10) = 62.8$ inches.

The Area = $(3.14)(10^2) = (3.14)(100) = 314$ square inches.

Your Exercise:

1. Find the circumference of a circle with radius of 5 inches.
2. Find the area of a circle with radius of 5 inches.
3. What is the diameter of a circle with area of 1256 square inches?